isaac computer science

Boolean algebra

Boolean algebra is a branch of mathematics in which algebraic expressions are made up of:

- Variables, which can represent the values 1 and 0
- The constants 1 and 0
- The operators AND, OR, and NOT

Boolean laws are statements of equivalence (called identities) between two Boolean expressions. These laws generally (but not always) follow rules that you will be familiar with from the standard rules of algebra; in this context AND ($\wedge$) can be considered as multiplication and OR ($\vee$) as addition.

Boolean expressions can be manipulated algebraically to form simpler expressions that implement the same logic. This may allow a simpler circuit to be produced with fewer logic gates. This will reduce the cost of the circuit, the amount of heat generated, and the processing time.

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Variables, constants, and operators

Digital circuits can have many inputs and you have seen that these are usually labelled with letters. Circuits can also have inputs that are permanently **on** (1) or **off** (0).

Boolean expressions are made up of variables and constants. It is important to remember that, whilst an expression can include many variables, the value of each of these variables can **only** be a 0 or a 1.

A digital circuit can include many types of logic gate. However, these gates combine the fundamental functions of Boolean logic: AND, OR, and NOT. This means that when you are asked to use Boolean algebra, you need only consider the operators $\wedge$, $\vee$, and $\neg$.

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Simple Boolean identities

Boolean identities are statements of equivalence. In this section, you will consider identities that refer to a single Boolean variable and the constants 0 and 1. Here, the Boolean variable is labelled A but the identity will apply to any Boolean variable. The identity will also apply to a statement that is made up of Boolean variables and constants.

Each identity is shown with a truth table that proves the equivalence. Where the values in a column are identical, the statements are equivalent. Some of these identities have special names and these are shown in brackets in the column heading.

Identity (Annulment)	Truth table

$A \wedge 1 = A$

A	1	$A \wedge 1$
0	1	0
1	1	1

Identity (Annulment)	Truth table

$A \vee 0 = A$

A	0	$A \vee 0$
0	0	0
1	0	1

Identity (Idempotent)	Truth table

$A \wedge A = A$

A	A	$A \wedge A$
0	0	0
1	1	1

Identity (Idempotent)	Truth table

$A \vee A = A$

A	A	$A \vee A$
0	0	0
1	1	1

Identity (Complement)	Truth table

$A \vee \neg A = 1$

A	$\neg A$	$A \vee \neg A$
0	1	1
1	0	1

Identity (Complement)	Truth table

$A \wedge \neg A = 0$

A	$\neg A$	$A \wedge \neg A$
0	1	0
1	0	0

Identity (Complement)	Truth table

$\neg\neg A = A$

A	$\neg A$	$\neg\neg A$
0	1	0
1	0	1

Identity	Truth table

$A \wedge 0 = 0$

A	0	$A \wedge 0$
0	0	0
1	0	0

Identity	Truth table

$A \vee 1 = 1$

A	1	$A \vee 1$
0	1	1
1	1	1

When you identify a simple identity, you can use it to simplify a Boolean expression. Find an identity to help

you simplify the following expression:

$(A \wedge B) \wedge \neg(A \wedge B)$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

GCSE **Commutative law**

The commutative law states that:

$X \wedge Y = Y \wedge X$

$X \vee Y = Y \vee X$

This means that the variables can be rearranged without affecting the logic of the statement.

Show with a truth table that $X \wedge Y = Y \wedge X$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

Show with a truth table that $X \vee Y = Y \vee X$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

GCSE **Associative law**

The associative law states that:

$X \wedge (Y \wedge Z) = (X \wedge Y) \wedge Z$

$X \vee (Y \vee Z) = (X \vee Y) \vee Z$

This allows for the removal and rearrangment of the brackets so long as the operators are the same.

Show using a truth table that $X \wedge (Y \wedge Z) = (X \wedge Y) \wedge Z$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

Show using a truth table that $X \vee (Y \vee Z) = (X \vee Y) \vee Z$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

GCSE **Absorption law**

The absorption law states that:

$X \vee (X \wedge Y) = X$

$X \wedge (X \vee Y) = X$

Show using a truth table that $X \vee (X \wedge Y) = X$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

Show using a truth table that $X \wedge (X \vee Y) = X$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

GCSE **Distributive law**

The distributive law states that:

$X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z)$

If you consider the $\wedge$ operator as multiplication, and the $\vee$ operator as addition, this is equivalent to expanding the brackets or factorisation (depending on which way you are working) in normal algebra.

It is also true that:

$X \vee (Y \wedge Z) = (X \vee Y) \wedge (X \vee Z)$

This is **not** like in normal algebra!

Prove with a truth table that $X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z)$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

Prove with a truth table that $X \vee (Y \wedge Z) = (X \vee Y) \wedge (X \vee Z)$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

A Level

De Morgan's laws

De Morgan's laws are named after Augustus De Morgan, a 19th-century British mathematician. De Morgan proved that:

$X \wedge Y = \neg(\neg X \vee \neg Y)$

$X \vee Y = \neg(\neg X \wedge \neg Y)$

You can read more about De Morgan's laws here.

Show with a truth table that $X \wedge Y = \neg(\neg X \vee \neg Y)$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

Show with a truth table that $X \vee Y = \neg(\neg X \wedge \neg Y)$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High