De Morgan's laws

Note: This page is not relevant for GCSE.

De Morgan's laws are named after Augustus De Morgan, a 19th-century British mathematician. De Morgan proved that:

$\neg(\neg A \vee \neg B) = A \wedge B$

$\neg(\neg A \wedge \neg B) = A \vee B$

De Morgan's laws are very useful when working with algebraic expressions that contain the logical NOT operator. As the NOT operator takes precedence over AND and OR operations, the ability to remove significant NOTs allows the expression to be manipulated more freely.

A Level

De Morgan's laws – three steps

There are three steps to applying De Morgan's laws:

1. Change the operator (AND to OR or vice versa)
2. Negate the variables on either side of the operator
3. Negate the whole statement and simplify

Let's see how this can be applied to show that $\neg(\neg A \vee \neg B) = A \wedge B$.

First take $\neg(\neg A \vee \neg B)$

Step 1: Change the operator

Change **OR** to **AND**:

$\neg(\neg A \wedge \neg B)$

Step 2: Negate the variables on either side of the operator

$\neg(\neg\neg A \wedge \neg\neg B)$

Step 3: Negate the whole statement and simplify

$\neg\neg(\neg\neg A \wedge \neg\neg B)$

At this stage, you can cancel out any "double-NOTs". This is because of the fundamental Boolean identity $\neg\neg X = X$, which you learnt previously.

You have now shown that:

$\neg(\neg A \vee \neg B) = A \wedge B$.

A Level

De Morgan's worked example (1)

Consider the expression:

$\neg(A \lor \neg B)$

Step 1: Change the operator

Change **OR** to **AND**:

$\neg(A \land \neg B)$

Step 2: Negate the variables on either side of the operator

Apply $\neg$ to A and $\neg B$

$\neg(\neg A \land \neg\neg B)$

Notice that B is now "double-NOTted"

Step 3: Negate the whole statement and simplify

$\neg(\neg(\neg A \land \neg\neg B))$

Notice that the whole statement is now "double-NOTted"

You can now cancel out any "double-NOTs". This is because of the fundamental Boolean identity $\neg\neg X = X$, which you learned previously.

$\neg A \land B$

Thus, you have shown that $\neg(A \lor \neg B)$ = $\neg A \land B$

A Level

De Morgan's worked example (2)

Consider the expression:

$\neg(\neg(A) \lor \neg(B \land A))$

Step 1: Change the operator

In applying step 1, you may perceive a problem. Which operator do you change? The important thing is that you must change **only one** operator at a time. Trying to change more than one (at the same time) will almost certainly result in a logic error.

In the given expression, the OR operator has $\neg(A)$ to its left and $\neg(B \land A)$ to its right. In the latter expression, the AND operator has B on its left and A on its right. You could tackle either operator, but it is recommended that you work to remove the outermost NOT first — remember the final step is to "negate the whole statement". This will result in you being able to remove the NOT that applies to the whole statement as it will be cancelled out.

Change **OR** to **AND**:

$\neg(\neg(A) \land \neg(B \land A))$

Step 2: Negate the variables on either side of the operator

Another issue arises when you consider how to apply step 2. You don't have a single variable to the right of the operator you just changed. To the right, is another statement: $\neg(B \land A)$. However, this statement can be treated as a single variable because it will evaluate as either 1 or 0 (as do **all** Boolean variables, statements, and expressions).

$\neg(\neg\neg(A) \land \neg\neg(B \land A))$

Step 3: Negate the whole statement and simplify

$\neg\neg(\neg\neg(A) \land \neg\neg(B \land A))$

You can now cancel out any "double-NOTs". This is because of the fundamental Boolean identity $\neg\neg X = X$, which you learned previously.

$A \land B \land A$

Thus, you have shown that:

$\neg(\neg(A) \lor \neg(B \land A))$ will simplify to $A \land B \land A$

At this point you will hopefully realise that the expression is still not fully simplified.

Can you carry out the final step of the algebraic simplification?

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low | Medium | High

A Level

De Morgan's worked example (3)

Consider the expression:

$\neg(\neg(A \vee B \wedge \neg B) \vee C \wedge A)$

Step 1: Change the operator

Here, you will change the OR operator that has $\neg(A \vee B \wedge \neg B)$ to its left and $C \wedge A$ to its right.

Change **OR** to **AND**:

$\neg(\neg(A \vee B \wedge \neg B) \wedge C \wedge A)$

Step 2: Negate the variables on either side of the operator

$\neg(\neg\neg(A \vee B \wedge \neg B) \wedge \neg(C \wedge A))$

Step 3: Negate the whole statement and simplify

$\neg\neg(\neg\neg(A \vee B \wedge \neg B) \wedge \neg(C \wedge A))$

You can now cancel out any "double-NOTs". This is because of the fundamental Boolean identity $\neg\neg X = X$, which you learned previously.

$A \vee B \wedge \neg B \wedge \neg(C \wedge A)$

Another application of De Morgan's laws

At this point, there are various approaches you can take, but you may spot that you can make another application of De Morgan's laws on this part of the expression:

$\neg(C \wedge A)$

Following the steps again, you can simplify this part of the expression to $(\neg C \vee \neg A)$

The whole expression at this stage is:

$A \vee B \wedge \neg B \wedge (\neg C \vee \neg A)$

Can you complete the algebraic simplification? It will help if you use some brackets to make sure that you don't forget the order of precedence.

$$A \vee (B \wedge \neg B) \wedge (\neg C \vee \neg A)$$

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

A Level

Proof with Venn diagrams

Another way of showing a proof for De Morgan's laws is to use Venn diagrams.

Consider the first law:

$A \wedge B = \neg(\neg A \vee \neg B)$

Study a Venn diagram that represents $A \wedge B$:

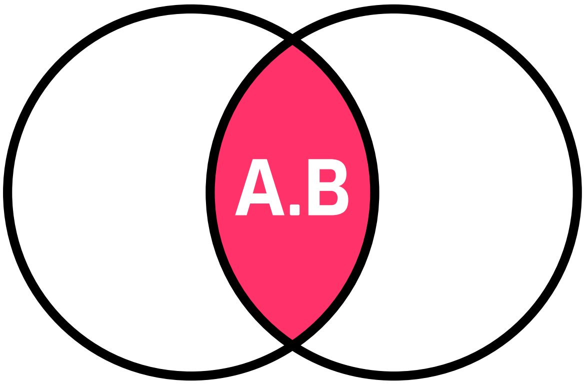

Figure 1: Venn diagram for A AND B

In the following diagrams, blue shows the area covered by NOT A, and green shows the area covered by NOT B.

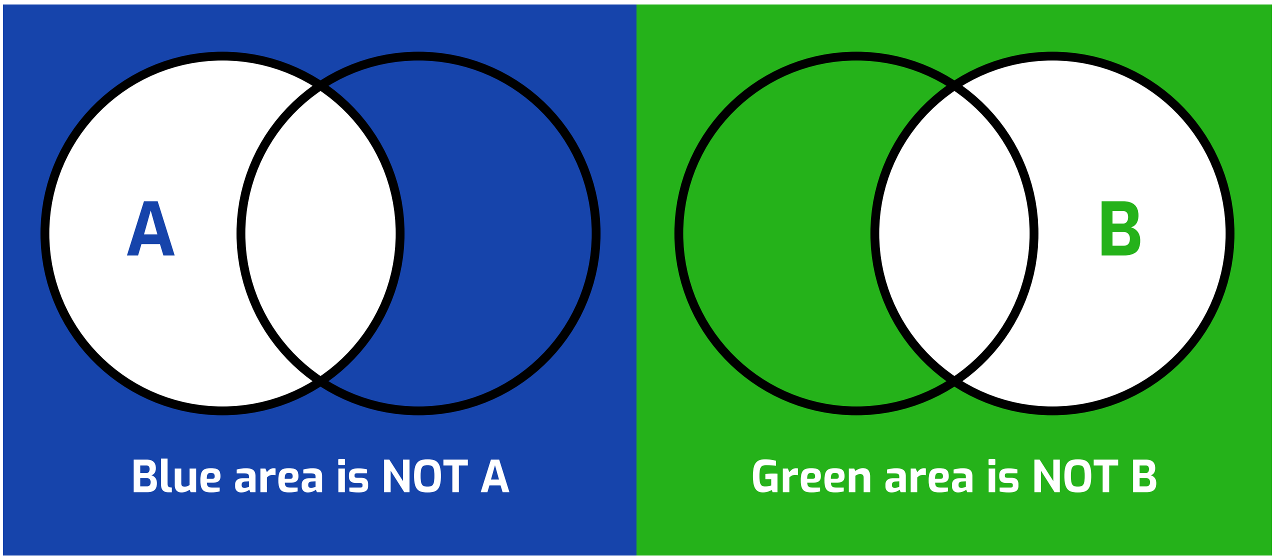

Figure 2: Venn diagrams showing NOT A and NOT B

In the final diagram, the yellow area covers the area that is NOT A OR NOT B.

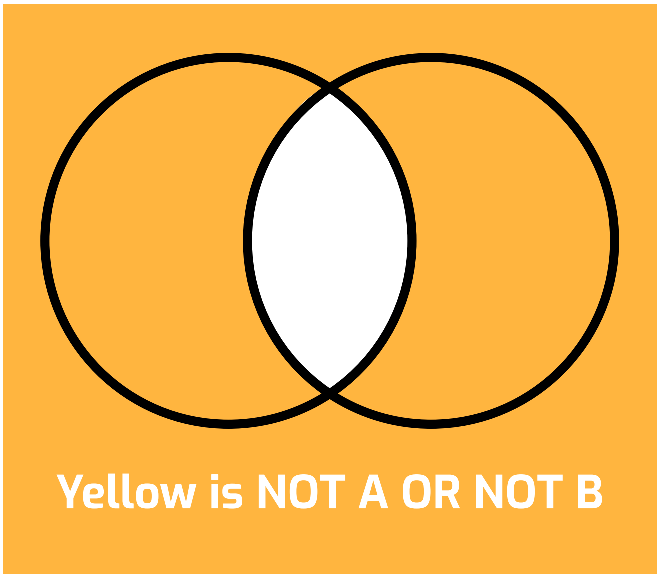

Figure 3: Venn diagram showing NOT A OR NOT B

This shows that $A \wedge B$ is the **opposite** of NOT A OR NOT B, meaning $\neg(\neg A \vee \neg B)$.