isaac computer science

Write a Boolean expression

A Boolean expression is one which when evaluated, will produce a Boolean value, in the same way as the statements explained in "What is Boolean logic?".

Digital circuits, truth tables, and problem statements can be represented using a Boolean expression. Using any one of those listed above, you can derive a Boolean expression to represent it, and vice versa. You could generate both a truth table or digital circuit from a Boolean expression. You could even break down an expression to formulate a problem statement from it.

GCSE **Write an expression from a circuit diagram**

In this first video, the construction of a Boolean expression from a logic circuit containing two logic gates is demonstrated.

Embedded YouTube video: https://www.youtube.com/watch?v=5ZvZ6nhDPME.
In this second video, the construction of a Boolean expression from a logic circuit containing three logic gates is demonstrated.

Embedded YouTube video: https://www.youtube.com/watch?v=62jpBUMca9Q.

Construct a Boolean expression from the cicuit diagram below, check back through the explainer videos if you need a reminder.

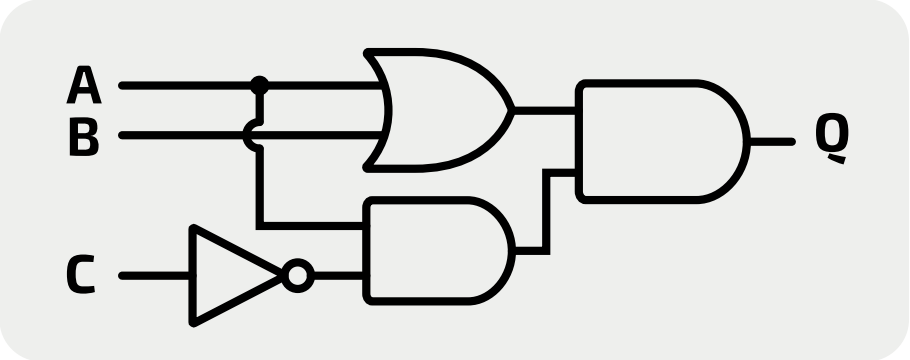

Figure 1: Circuit diagram

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

GCSE **Write an expression from a simple truth table**

Sometimes you may be given a truth table and be asked to write an expression.

Consider the following truth table:

Inputs		Output
A	B	Q
0	0	1
0	1	1
1	0	0
1	1	1

The first thing to consider is that the truth table may match a basic two-input Boolean function. There are three basic functions (AND, OR, and NOT), and at an advanced level there is also NAND and NOR. It is useful to memorise their truth tables, if you can, for situations like this. In this case, the truth table does **not** match a basic function.

Now you must consider **only** the lines that result in an output of 1 because an expression defines the combination of inputs that result in an output of 1. This is very important — you don't need to worry about the 0s.

In this example, a 1 is output:

- Whenever A is 0 — this logic is $\neg A$
- Whenever B is 1 — this logic is B

Therefore, the expression for this circuit is $Q = \neg A \vee B$

Create the truth table for the expression $\neg A \vee B$ to show that the outputs are the same as for the table shown above.

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

A Level

Write an expression from a circuit diagram

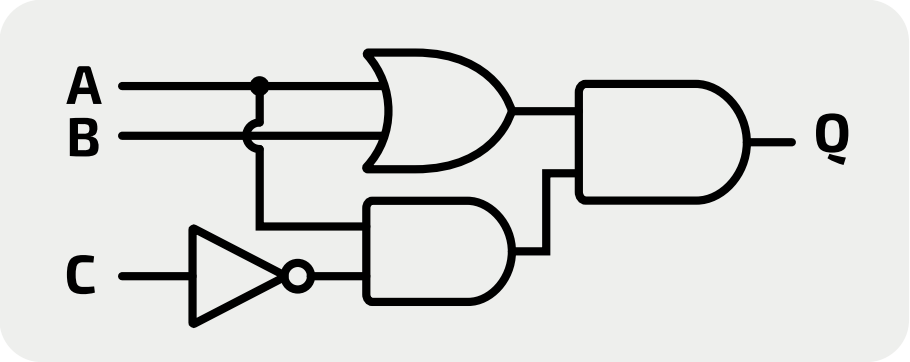

Figure 1: Circuit diagram

Consider the circuit diagram shown above in **Figure 1**.

Take each gate in turn, working from left to right, and write an expression that represents the logic of that part of the circuit. In the figures below, the expressions have been written using words for the operators.

The NOT gate (blue box)

Consider the **NOT** gate:

- The input is C
- The output is $\neg C$

Write $\neg C$ on the output line.

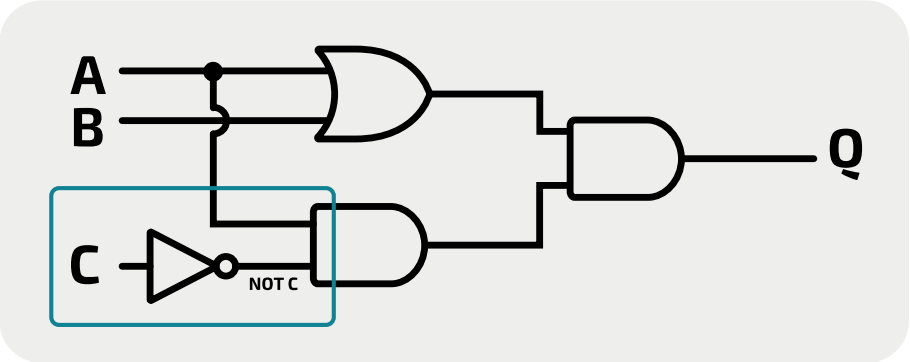

Figure 2: NOT C

The AND gate (pink box)

Consider the first **AND** gate:

- One input is A
- One input is $\neg C$
- The output is $A \wedge \neg C$

Write $A \wedge \neg C$ on the output line.

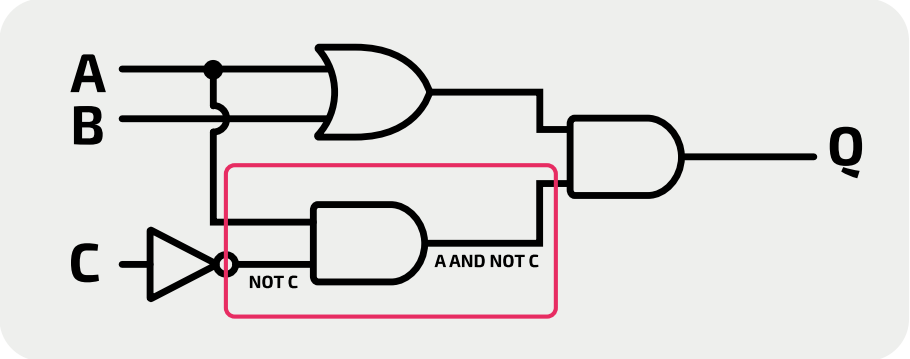

Figure 3: A AND NOT C

The OR gate (orange box)

Consider the **OR** gate:

- One input is A
- One input is B
- The output is $A \vee B$

Write $A \vee B$ on the output line.

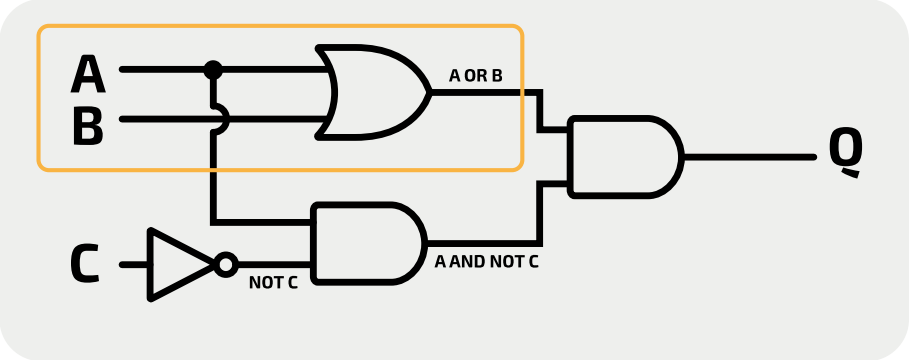

Figure 4: A OR B

The AND gate (purple box)

Consider the second **AND** gate:

- One input is $A \vee B$
- One input is $A \wedge \neg C$
- The output is $(A \vee B) \wedge (A \wedge \neg C)$

Write $(A \vee B) \wedge (A \wedge \neg C)$ on the output line.

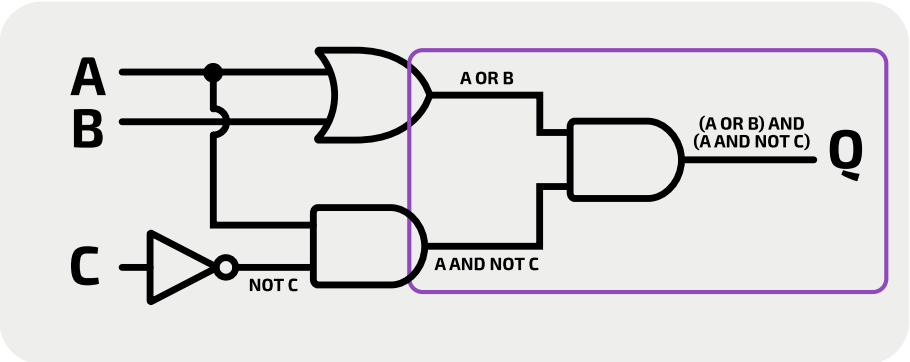

Figure 5: (A OR B) AND (A AND NOT C)

This final output is the expression for the circuit — $(A \vee B) \wedge (A \wedge \neg C)$.

- You don't actually need the brackets around $A \wedge \neg C$ but they do no harm and may help make the expression easier to read.
- Being able to break up the expression in this way is helpful if you need to draw a truth table for part of a circuit diagram.

A Level **Write an expression from a problem statement**

Consider the following problem statement:

Sam and Zac live in a house with a smart heating system. The system will turn the heating on if either of them is at home and the temperature drops below $18℃$. The smart system can be switched off completely using an override button.

- The temperature is measured by a sensor that returns 1 if the termperature drops below $18℃$ and otherwise returns 0.
- Sam's and Zac's mobile phones connect to the WiFi when they are home. If a phone is connected to the WiFi, it can be sensed by the system — 1 means that the phone is connected to the WiFi, 0 means that the phone is not connected to the WiFi.
- The override button returns 1 if the system is switched off (and otherwise returns 0).
- The heating is switched on and off by an actuator. When the actuator is 1, the heating will be on.

Identify and label the inputs

There are four inputs to the system:

- A temperature sensor (T)
- A sensor that detects if Sam's phone is connected to the WiFi (S)
- A sensor that detects if Zac's phone is connected to the WiFi (Z)
- An override button (B)

Identify and label the output

The output of the system is the actuator (H).

Break down the logic from the problem statement

The heating will turn on if:

- The override button (B) is not switched on

AND

- Sam is home (S) OR Zac is home (Z) AND the temperature is below $18℃$ (T)

Express the logic using only the variables

- NOT B

AND

- S OR Z AND T

This last part needs some brackets. The logic is (S OR Z) AND T. The brackets are needed because the order of operator precedence would mean that, without the brackets, the expression would be evaluated as S OR (Z AND T).

Write the final expression

This should now be straightforward. The final statement is:

(NOT B) AND ((S OR Z) AND T)

Using symbols, the expression is:

$\neg B \wedge ((S \vee Z) \wedge T)$

Check with a truth table

It is useful to check your work with a truth table. Because there are 4 inputs, the table will have 16 rows. If you are not confident in setting the table out, look at this page.

Enter exposition here

Click a button to show the answer

What is your level of confidence that your own answer is correct?

Low Medium High

A Level

Write an expression from a more complex truth table

Now consider a truth table with three inputs. In this truth table, the rows have been numbered so that they can be referred to in the explanation that follows.

	Inputs			Output
	A	B	C	Q
1.	0	0	0	0
2.	0	0	1	0
3.	0	1	0	1
4.	0	1	1	1
5.	1	0	0	1
6.	1	0	1	0
7.	1	1	0	1
8.	1	1	1	1

Focus on the rows where the output is 1; these rows have been shaded in the table. You are looking for a consistent pattern of input.

The output is always 1 when B = 1, so one part of the expression is just B. This will account for rows 3, 4, 7, and 8 in the truth table. However, there is another combination of inputs on line 5 that also produces an output of 1. Thus, the final expression will be $B \vee ?$ — where $?$ is a statement that uniquely covers the logic of row 5. This row is:

Inputs			Output
A	B	C	Q
1	0	0	1

The logic that covers this row is $A \wedge \neg B \wedge \neg C$. However, you already know that B, on its own, is part of the expression, so you can ignore B — the part you want is $A \wedge \neg C$. If you are not conviced, check the truth table. You can see that line 7 also satisfies the logic $A \wedge \neg C$ to produce a 1 as output. Now you can see that $\neg B$ is irrelevant.

The expression that can be derived from the truth table is:

$Q = B \vee (A \wedge \neg C)$

What is the Boolean expression for the following truth table?

Inputs			Output
A	B	C	Q
0	0	0	0
0	0	1	1
0	1	0	0

Inputs			Output
A	B	C	Q
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

A Level

Using the Boolean editor

Low Medium High

Some of our questions require you to use the Boolean editor. Make sure you are signed in and have your preferences set correctly so you get the right set of symbols. The video below explains how to use the editor.

Embedded YouTube video: https://www.youtube.com/watch?v=4EccK8HNwRM.